Chapter 4: Force and Motion

We are skipping:

Chapter 3: Vectors and Coordinate Systems

Students are nevertheless responsible for its contents (grade 11 math) and to be able to do the type of problems in its end-of-chapter (EOC) problem sets.

We go directly to Chapter 4, an introduction to dynamics.

Dynamics is the branch of physics that seeks the reasons for motion: forces.

kinematics + dynamics = mechanics
Chapter 4: Force and Motion

Definition: A force

• is a push or a pull on an object,
• is a vector quantity (both direction and magnitude), and
• requires an agent (something that does the pushing or pulling).

A force can either be a contact force (e.g., normal force, string tension, etc.) or a long-range force (e.g., gravity, electrical forces, etc., known in antiquity as an action at a distance).

This semester, the only long range force we’ll use is gravity. All other forces will be contact forces.
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Representing forces as vectors:
Whether a force is:
• a contact force or long-range;
• pushing on the object or pulling,
always represent a force as a vector with:
• its tail anchored to the object being forced;
• its tip pointing in the direction of the force;
• labels for both the force and the object.
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Forces are measured in **Newtons** (N) (we’ll come back to this)

**Adding forces as vectors**

If \( \vec{F}_1 = (3, 4) \text{ N} \) and \( \vec{F}_2 = (0, -5) \text{ N} \)

\[ \vec{F}_{\text{net}} = (3+0, 4-5) = (3, -1) \text{ N} \]

**Force components:**

\[ F_x = F \cos(\theta) \]
\[ F_y = F \sin(\theta) \]
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4.2 Force “Catalogue”

1. **Weight** is the force of gravity acting downward. It is a **long-range force**, and its agent is the *entire* earth.

   \[ \vec{w} = m\vec{g} = -mg \hat{y} \]

   The weight force pulls the box down.

2. A **spring force** acts in a direction opposite to the distortion (\(\Delta x\)) of the spring (compressed or stretched). It is a **contact force**.

   \[ \vec{F}_{sp} = -k\Delta x \]

   A compressed spring exerts a pushing force on an object.

   A stretched spring exerts a pulling force on an object.
3. **Tension** (in a rope or wire) always acts in the direction of the rope. It is a contact force and represented by: $\vec{T}$

4. A **normal force** is exerted by a surface on an object pressing against that surface. It is a contact force, always acts perpendicular (normal) to the surface, and is represented by: $\vec{n}$

$n$ does not always equal $mg$!!!
5. **Resistive forces** (all contact)

a) *static friction*, \( f_s \leq \mu_s n \), prevents motion along a surface and points in the direction opposite to the motion it prevents.

b) *kinetic friction*, \( f_k = \mu_k n \), resists motion along a surface, and points in the direction opposite to the motion.

c) *drag* is kinetic friction caused by motion through air.

\[ D = bA v^2 \]
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6. Other forces

- **thrust**: $\vec{F}_{\text{thrust}}$ (contact)
- **electrical force**: $\vec{F}_E = q\vec{E}$ (long-range)
- **magnetic force**: $\vec{F}_M = q\vec{v} \times \vec{B}$ (long-range)
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Summary of forces

<table>
<thead>
<tr>
<th>name</th>
<th>formula</th>
<th>agent</th>
<th>type</th>
</tr>
</thead>
<tbody>
<tr>
<td>gravity</td>
<td>$\vec{w} = m\vec{g}$</td>
<td>earth</td>
<td>long-range</td>
</tr>
<tr>
<td>spring force</td>
<td>$\vec{F}_{sp} = -k\Delta\vec{x}$</td>
<td>spring</td>
<td>contact</td>
</tr>
<tr>
<td>Tension force</td>
<td>$\vec{T}$</td>
<td>rope, wire</td>
<td>contact</td>
</tr>
<tr>
<td>normal force</td>
<td>$\vec{n}$</td>
<td>surface</td>
<td>contact</td>
</tr>
<tr>
<td>static friction</td>
<td>$f_s \leq \mu_s n$</td>
<td>surface</td>
<td>contact</td>
</tr>
<tr>
<td>kinetic friction</td>
<td>$f_k = \mu_k n$</td>
<td>surface</td>
<td>contact</td>
</tr>
<tr>
<td>air drag</td>
<td>$D = bAv^2$</td>
<td>air</td>
<td>contact</td>
</tr>
<tr>
<td>Thrust</td>
<td>$\vec{F}_{thrust}$</td>
<td>hot gas</td>
<td>contact</td>
</tr>
</tbody>
</table>

$m = \text{gravitational mass} \quad g = 9.80 \text{ m s}^{-2}$

$\Delta x = \text{displacement} \quad \mu_s = \text{coefficient of static friction (unitless)}$

$\mu_k = \text{coefficient of kinetic friction (unitless)} \leq \mu_s \quad A = \text{cross-sectional area}$

$k = \text{spring constant (N m}^{-1}) \quad b = 0.25 \text{ kg m}^{-2} \text{ for air}$
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Pre-Newtonian/Galilean mechanics: Motion according to Aristotle

The natural state of all objects is to be at rest. To keep things moving, one must always apply a force. Thus, once the horse stops pulling the cart, the cart comes to rest.
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Pre-Newtonian/Galilean mechanics: Motion according to Aristotle

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Excuse me, Mr. Aristotle, but then what keeps the arrow moving once it’s been launched from the bow?
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Excuse me, Mr. Aristotle, but then what keeps the arrow moving once it’s been launched from the bow?
I see…

Well then what about this? If I let go of a stone---I don’t even need to give it a push---it gathers speed all on its own accord. If rest is the stone’s natural state, why does it start moving from rest?
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Earth, water, wind and fire---elements seek out their own kind. Solid rock seeks out solid earth.
I see…

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Hmmm, fascinating…

So why does the fire in my firepit not jump over to join the fire in yours?

Why does a stick float and not join the solid bottom?

Why, why, why?
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Why, why, why?

Go home, Isaac…
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The three laws of Sir Isaac Newton (1687)

1. A body’s velocity will not change if and only if there is no net force acting on it.

2. The net force on a body is equal to the product of the body’s mass and acceleration: \( \Sigma F = F_{\text{net}} = ma \).

3. When two bodies interact, the forces exerted by each body on the other are always equal in magnitude and opposite in direction.

*If greatness is measured by the magnitude and scope of impact on humanity, these are among the greatest utterances ever uttered by a human.*
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Other versions you may have heard of…

1. *Law of Inertia*: A body at rest tends to stay at rest and a body in motion tends to stay in motion unless acted upon by a *net* external force.

2. $F = ma$ and you can’t push on a rope (*the first-year engineer’s dictum*).

3. For every action, there is an equal and opposite reaction.
Chapter 4: Force and Motion

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Or, we could just go with what Newton himself wrote...

*Lex I.* Corpus omne perseverare in statu suo quiescendi vel movendi uniformiter in directum, nisi qua’tenus a viribus impressis cogitur statum illum mutare.

*Lex II.* Mutationem motus proportionalem esse vi motrici impressae, et fieri secundum lineam rectam qu’a vis illa imprimitur.

*Lex III.* Actioni contrariam semper et aequalem esse reactionem: sive corporum duorum actiones in se mutuo semper esse aequales et in partes contrarias dirigi.

(see the [website](http://example.com) for a widely-accepted translation)
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Quotations from/about Sir Isaac...

*If I have seen farther, it is by standing upon the shoulders of giants.*

(in a letter to Robert Hooke, another well-known natural philosopher of the day, who was a diminutive man with a hunchback and for whom Newton held considerable disdain.)

*I do not know what I may appear to the world, but to myself I seem to have been only like a boy playing on the sea-shore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me.*

*In the absence of any other proof, the thumb alone would convince me of God's existence.*

His epitaph: *Who, by vigor of mind almost divine, the motions and figures of the planets, the paths of comets, and the tides of the seas first demonstrated.*

*Nature and nature's laws lay hid in night; God said "Let Newton be" and all was light.* (Alexander Pope)
Newton’s 1st Law (*Lex I*) is really a special case of the 2nd (*Lex II*). After all, if $\mathbf{F} = 0$, $\mathbf{a} = 0$ and velocity must remain constant.

You can be sure that Newton himself realised this. So why do you suppose he stated *Lex I* separately?
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Newton’s 1\textsuperscript{st} Law \((Lex\ I)\) is really a special case of the 2\textsuperscript{nd} \((Lex\ II)\). After all, if \(\vec{F} = 0\), \(\vec{a} = 0\) and velocity must remain constant.

You can be sure that Newton himself realised this. So why do you suppose he stated \textit{Lex I} separately?

- a deliberate counter to the prevailing Aristotlean view that to keep things in motion one must continually apply a force.

- in this “opener”, he is telling all who read to cast aside all they \textit{thought} they knew; he is about to enlighten them...

We will concentrate on \textit{Lex II}, the quantitative law, upon which all “classical physics” is based. Newton’s 3\textsuperscript{rd} Law \((Lex\ III,\ action\text{-}reaction)\) will come later.
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The unit for force is \textit{Newton} (N).

Since $\vec{F}_{\text{net}} = m\vec{a}$, 1 N $= 1$ kg m s$^{-2}$ is the SI equivalent.

Force is a vector. To find the \textit{net force}, one must add the vectors.

$\vec{F}_{\text{net}} = m\vec{a}$ means the net force vector is in the \textit{same direction} as the acceleration vector.

The ratio of the magnitudes, $F_{\text{net}}/a$, is the \textit{inertial mass}, $m$, of the object (a.k.a., \textit{inertia}, or \textit{mass}).
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Clicker question 4.1

In which direction does the object accelerate?

(a) \( \vec{a} \)  
(b) \( \vec{a} \)  
(c) \( \vec{a} \)  
(d) \( \vec{a} \)  
(e) \( \vec{a} \)
Chapter 4: Force and Motion

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acceleration is in the same direction as the net force.
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Clicker question 4.2

Two forces, \( \vec{F}_1 \) and \( \vec{F}_2 \), are exerted on an object as shown to the left. What third force, \( \vec{F}_3 \), would make the net force point to the left?

(a) \( \vec{F}_3 \)

(b) \( \vec{F}_3 \)

(c) \( \vec{F}_3 \)

(d) \( \vec{F}_3 \)
Chapter 4: Force and Motion

Clicker question 4.2

ended here, 23/9/08
Inertial Reference Frames
- a coordinate system in which Newton’s Laws are valid

**Key point:** if no forces are applied to an object, object should not accelerate.

**Thought experiment:** Place a ball on the level floor of an airplane just before it starts to taxi. The ball stays in place. Jets roar into action, what happens to the ball? It spontaneously accelerates to the back of the plane, even though no forces were applied to it.
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Continuing with our thought experiment: The ball is put back on the floor when the plane reaches cruising level and speed (no turbulence). What happens now? The ball stays in place.

Conclusions: Newton’s Laws (no acceleration without forces) are valid only in frames of reference that are not themselves accelerating. An inertial reference frame is one that is not accelerating with respect to the distance stars.

Examples of non-inertial reference frames:
- a jet gaining speed before take-off
- a car turning around a sharp corner

Examples of inertial reference frames:
- a car moving at constant speed over a smooth straight road
- this room (despite rotating Earth, etc.)
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**Identifying forces**

To apply Newton’s 2nd Law (Lex II), we must identify all relevant forces correctly.

1. Identify the system. Here the system is the skier.
2. Draw a picture of the situation.
3. Draw a closed curve around the system.
4. Locate the points where the environment touches the system. Here the rope and the ground touch the skier.
5. Name and label each contact force. The rope exerts a tension force and the ground exerts both a normal and a kinetic friction force.
6. Name and label long-range forces. Weight is the only one.
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Pitfalls:

1. Fictitious forces
   - arise from observing system in a non-inertial frame.
     e.g., “centrifugal force”

2. “Made-up” forces
   - “kicking force”, “accelerating force”, “force of motion”

Rules of thumb:

1. All forces should come from the catalogue (e.g., a “pushing force” is often a “normal force”).
2. All forces should have an identified “agent”; something currently responsible for the force.
3. For contact forces, forces are exerted at the point of contact.
4. For long-range forces, forces are exerted at the centre of mass.
Clicker question 4.3

You’ve just kicked a stone and it is now sliding (not bouncing) across the sidewalk in front of you. Which of these forces or combination of forces act on the stone?

a) gravity, acting downward;

b) the normal force from the sidewalk, acting upward;

c) the force of the kick, acting in the direction of motion;

d) friction, acting opposite to the direction of motion;

e) a, b, c only;

f) a, b, d only;

g) a, c, d only;

h) all
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e) a, b, c only; f) a, b, d only; g) a, c, d only; h) all
Clicker question 4.4

Toss a ball straight up in the air. After the ball has left your hand but before it reaches its highest point, what forces are acting on the ball? (Do not ignore air drag).

a) its weight acting downward;

b) air drag acting upward;

c) air drag acting downward;

d) the force from the throw acting upward;

e) a and b only;

f) a and c only;

g) a, b, and d only;

h) a, c, and d only
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g) a, b, and d only;  h) a, c, and d only
4.7 Free-body diagrams (FBD)

An FBD is a diagram suitable to applying Newton’s 2nd Law showing:
- the object reduced to a point;
- a coordinate system with the object at the origin;
- all forces acting on the object drawn with their tails on the object;
- A single “off-dot” vector representing the net force (or acceleration).

One pictorial representation and two physical representations (motion diagram, FBD) of the same problem.
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Coordinate system should be centred on particle. It need not be horizontal-vertical (as in the example below).

To help identify forces properly, draw a “control surface” tightly around the object. The only contact forces affecting the object are those directly penetrating the control surface (tension & normal force in example below).
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eexample 1: A mass $m$ is pushed with force $\vec{F}$ causing both $m$ and $M$ to accelerate across the floor. Including frictional forces with the floor but ignoring air drag, create an FBD for mass $M$. 

![Diagram of mass $m$ being pushed by force $\vec{F}$, with mass $M$ on the floor.]
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Construct a coordinate system, and represent $M$ as a point at its origin.
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long-range force(s)

weight: \( \vec{w} \)
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normal force from the floor: $\vec{n}_{\text{Floor}}$

…and identify the contact forces that penetrate the control surface.
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![Diagram showing forces and friction on M and m](image-url)
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normal force from mass $m$: $\vec{n}_m$
example 1: A mass $m$ is pushed with force $\vec{F}$ causing both $m$ and $M$ to accelerate across the floor. Including frictional forces with the floor but ignoring air drag, create an FBD for mass $M$.

Note that $\vec{F}$ does not act directly on $M$ (it doesn’t penetrate the control surface), and thus is not included in the FBD.
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Clicker question 4.5

An elevator suspended by a cable is moving upwards, and slowing to a stop. Which FBD is the correct one?

(a)  
(b)  
(c)  
(d)  

$\vec{T}$  
$\vec{w}$  
$\vec{F}_{\text{elevator}}$
Chapter 4: Force and Motion

Clicker question 4.5

An elevator suspended by a cable is moving upwards, and slowing to a stop. Which FBD is the correct one?
Clicker question 4.6

A heavy box sits on the flatbed of a truck which accelerates to the right with acceleration \( \hat{a} \). If the box does not slip, which FBD best describes the dynamics of the box?
A heavy box sits on the flatbed of a truck which accelerates to the right with acceleration $\vec{a}$. If the box does not slip, which FBD best describes the dynamics of the box?

(a) (b) (c) (d)

no such thing as $F_{\text{truck}}$!